

Accelerated quarks and energy loss in confinement theory

Kazuo Ghoroku^{a*}, Kouki Kubo^{a†},

^aFukuoka Institute of Technology, Wajiro, Higashi-ku
Fukuoka 811-0295, Japan

Abstract

We study the energy loss rate (ELR) of the accelerated quark in terms of the holographic models for the two different motions, linear acceleration and uniform rotation. They are examined by two different non-conformal models with confinement. We found in both models that the value of ELR is bounded from below by the string tension of the linear confinement potential between quark and anti-quark. The lower bounds of ELR are independent of the types of the motion of the quark. They are determined by the string tension at the world sheet horizon of the model. These results are obtained when the model has the diagonal background metric.

*gouroku@dontaku.fit.ac.jp

†kkubo252@gmail.com

1 Introduction

The radiation from accelerating quarks is basic and interesting problem of the Yang-Mills gauge theory. It has a great variety of aspects according to the motion of the object in various stage of the interaction including the strong coupling regime. It is therefore very difficult to describe theoretically the radiation from the accelerating object. The energy loss of the accelerated quarks are determined by various different mechanisms of rich dynamics of the Yang-Mills theory. The approach from the lattice simulation is also inadequate to understand this time dependent phenomenon.

On the other hand, the gauge/gravity duality helps us to tackle this problem. This approach is very powerful in solving the dynamics in the strong coupling regime of the gauge theory. A moving quark, which obeys the dynamics of 4D gauge theory with strong coupling constant, can be described by a classical string solved in 5D gravity. This string ends on the boundary of 5D bulk and the end points correspond to the positions of the quark and anti-quark. We can obtain informations about the radiation from this quark when it is accelerated by some accidental force.

The charged particles like quarks radiate when they are accelerated as is well-known. This radiation is related to the world sheet horizon of the string corresponding to this accelerated quark. The ELR (Energy Loss Rate) of the quark due to this radiation is obtained in terms of the world-sheet momentum Π_t^σ as follows [1]-[4],

$$\frac{dE}{dt} \equiv \Pi_t^\sigma \equiv \frac{\delta S}{\delta(\partial_\sigma X^0)} \quad (\text{at world sheet horizon}), \quad (1)$$

where X^0 is the time-component of the string embedded function. S denotes the string action, and the horizon on the world sheet is defined by the zero-point of the induced metric for the string such that $g_{\tau\tau} = 0$. The formula (1) has been given by Xiao [1] by considering the energy of the string behind the world sheet horizon as the lost energy due to the radiation. Although this formula has been shown in the AdS case, we extend it to the other bulk background. On the other hand, the ELR for uniform rotating quark has been given in Ref. [3] for $\mathcal{N} = 4$ SYM theory and also for non-conformal and confining D4/ S^1 model in Ref. [4]. In these cases, the ELR is given as a coordinate independent form, so the above formula is also useful.

In the Ref. [4], it is pointed out that there is a lower bound of the ELR for the confinement case, and then the radiation is related to the jet of glueballs which have the mass gap given by the confinement mass scale. In this model, we could find the same lower bound of ELR for the case of the uniform accelerating quark. Here we extend this analysis to the other non-conformal and confining Yang-Mills theory [5, 6] for the uniform accelerating and the uniform rotating quarks with a constant angular velocity.

Then we could find the ELR lower bound being common to the both types of acceleration. This is not accidental. The lower bound is given by the tension, which is denoted as τ_{QCD} , of the linear potential between the quark and anti-quark in both models. This fact could be understood as follows. In order to keep the accelerated motion of the quark considered here, we need an external force F . It should overcome at

least the force coming from the linear potential which supports the quark confinement. So we consider as $F \geq \tau_{\text{QCD}}$. Furthermore, this force would provide the energy to the quark and the stretching string. The radiation part of the provided energy would be evaluated at the world sheet horizon as the string energy elongated by the tension at this point, $\tau(z)$, within a unit time as

$$\frac{dE}{dt} = \tau(z)v|_{\text{at horizon}} = \tau(z)|_{\text{at horizon}} \geq \tau_{\text{QCD}}, \quad (2)$$

where $v(=1)$ denotes the velocity of the string at the world sheet horizon. The details are given below for each accelerated motion in the Sec. 3.

The outline of this paper is as follows. In the next section, the solutions of supergravity, dAdS and $D4/S^1$, are given as the dual of two SYM theories with confinement. In the Sec.3, the ELRs for linear acceleration and uniform rotation are given for dAdS model. In Sec. 5, the same analysis is given for $D4/S^1$. In both cases, we could find the lower bound of the ELR as shown by (2). The summary and discussions are given in the final section.

2 Model

In this work, we consider the case of diagonal background metric G_{MN} . Then we can write the action and the canonical momentum density associated with the string as

$$S = -\frac{1}{2\pi\alpha'} \int d\tau d\sigma \sqrt{-g_{\tau\tau}g_{\sigma\sigma} + g_{\tau\sigma}^2},$$

$$\Pi_t^\sigma = -\frac{1}{2\pi\alpha'} \frac{-g_{\tau\tau}\partial_\sigma X^0 G_{tt} + g_{\tau\sigma}\partial_\tau X^0 G_{tt}}{\sqrt{-g_{\tau\tau}g_{\sigma\sigma} + g_{\tau\sigma}^2}},$$

where $g_{ab} \equiv \partial_a X^M G_{MN} \partial_b X^N$ ($a, b = \tau, \sigma$) is the induced metric and $X^M(\tau, \sigma)$ is the string embedding function. Choosing the gauge $\tau = t \equiv X^0$, we have the ELR which defined by Eq. (1),

$$\frac{dE}{dt} = \frac{|G_{tt}|}{2\pi\alpha'} \Big|_{z=\bar{z}} \geq \tau_{\text{QCD}}, \quad (3)$$

where \bar{z} denotes the world sheet horizon, $g_{\tau\tau}(\bar{z}) = 0$. *

We examine following two different holographic confining gauge theory model.

deformed AdS by dilaton (dAdS model) One of the models we use is very similar to the AdS model but it is not conformal [5, 6]. The metric is written as

$$\begin{aligned} ds^2 &= G_{MN} dX^M dX^N \\ &= e^{\Phi/2} \frac{L^2}{z^2} \left(-dt^2 + d\vec{x}^2 + dz^2 \right), \quad e^\Phi = 1 + qz^4 \end{aligned} \quad (4)$$

*Since $\tau_{\text{QCD}} = \frac{\min(|G_{tt}|)}{2\pi\alpha'}$, the confinement property of the gauge theory (non-zero value of τ_{QCD}) directory indicates the existence of the lower bound of the ELR.

The difference from AdS space-time is the part including constant q which breaks the conformal symmetry. Then the theory gets the typical energy scale, $q^{1/4}$, as the free parameter.

This parameter q plays an important role in this model: It makes the model confinement. We can see that the quark-anti quark potential proportional to the distance between the quarks times \sqrt{q} . Thus the model with any finite q is confinement.

D4/ S^1 model Another model is Witten's D4/ S^1 model which is very well known [8]. The model constructed by N D4 branes on a circle, and it gives the metric as follows,

$$ds^2 = \left(\frac{u}{l}\right)^{3/2} \left(-dt^2 + d\vec{x}^2 + f dx_4^2\right) + \left(\frac{l}{u}\right)^{3/2} \left(\frac{du^2}{f} + u^2 d\Omega_4^2\right), \quad f = 1 - \left(\frac{u_k}{u}\right)^3, \quad (5)$$

where l is the typical length scale of this model.

3 dAdS model

3.1 Linear acceleration(uniform acceleration)

First we consider the uniform accelerating quark in dAdS model. We parameterize the string embedding function as

$$X^M = (t, x(t, z), 0, 0, z).$$

Here we choose $(\tau, \sigma) = (t, z)$. Then the action and the equation of motion are written as [2]

$$S = -\frac{1}{2\pi\alpha'} \int dt dz e^{\Phi/2} \frac{L^2}{z^2} \sqrt{1 - \dot{x}^2 + x'^2},$$

$$\partial_t \left(e^{\Phi/2} \frac{L^2}{z^2} \frac{-\dot{x}}{\sqrt{1 - \dot{x}^2 + x'^2}} \right) + \partial_z \left(e^{\Phi/2} \frac{L^2}{z^2} \frac{x'}{\sqrt{1 - \dot{x}^2 + x'^2}} \right) = 0. \quad (6)$$

If we assume that the function x can be written as[†]

$$x = \sqrt{t^2 + g(z)}, \quad (7)$$

the equation (6) is reduced to

$$g'' - 2 \left(\frac{1}{4} g'^2 + \left(\frac{1}{z} \frac{1}{1 + qz^4} g' + 1 \right) \left(g + \frac{1}{4} g'^2 \right) \right) \frac{1}{g} = 0,$$

and the induced metric on the world sheet becomes

$$g_{ab} = e^{\Phi/2} \frac{L^2}{z^2} \begin{pmatrix} \frac{-g(z)}{x^2} & \frac{1}{2} \frac{tg'}{x^2} \\ \frac{1}{2} \frac{tg'}{x^2} & \frac{t^2 + g + \frac{1}{4} g'^2}{x^2} \end{pmatrix}.$$

[†]We can identify $g(0)$ with the reciprocal of the square of the acceleration of the quark, namely,

$$g(0) = \frac{1}{a^2}$$

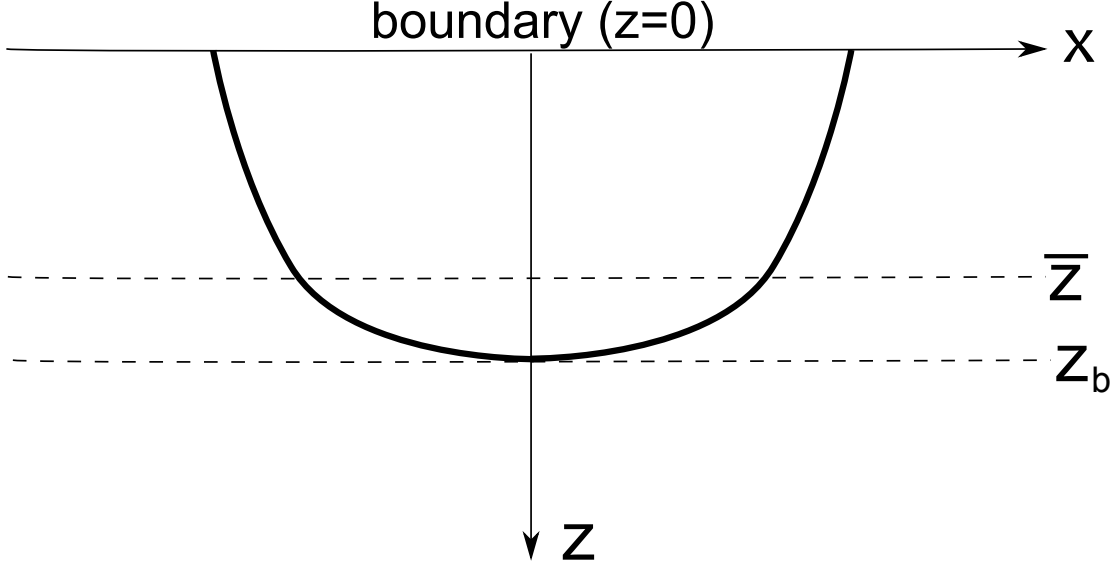


Fig. 1: The conceptual sketch of the linearly accelerating string. There is an induced metric horizon at $z = \bar{z}$. This string can be regarded as the pair of the quark and the antiquark.

The conceptual sketch of this string expressed by (7) is shown in Fig. 1.

The ELR is

$$\frac{dE}{dt} = \frac{1}{2\pi\alpha'} \frac{L^2}{\bar{z}^2} \sqrt{1 + q\bar{z}^4} \geq \frac{L^2\sqrt{q}}{2\pi\alpha'}. \quad (8)$$

There is the lower bound of this quantity, $\frac{L^2\sqrt{q}}{2\pi\alpha'}$, in the limit $\bar{z} \rightarrow \infty$. Since the horizon \bar{z} goes far away in this limit, the limit corresponds to the small acceleration limit. It can be seen in Fig. 2.

We notice here that the above lowest value of ELR is equivalent to the tension parameter $\tau_{\text{QCD}} (= \frac{L^2\sqrt{q}}{2\pi\alpha'})$ [7], which is defined by the quark and anti-quark potential V as $V = \tau_{\text{QCD}} r$ for large r , where r denotes the distance between the quark and anti-quark. This is expected as mentioned above through equation (2).

The above equation (8) is obtained by the consideration given in the introduction. The ELR is evaluated as the energy of the part pulled by the tension at $z = \bar{z}$ for a unit time in the direction of x . It is given as

$$\frac{dE}{dt} = \frac{1}{2\pi\alpha'} \int_0^1 dx \frac{L^2}{\bar{z}^2} \sqrt{1 + q\bar{z}^4} = \frac{1}{2\pi\alpha'} \frac{L^2}{\bar{z}^2} \sqrt{1 + q\bar{z}^4}. \quad (9)$$

From this, we find the force given in (2) as the tension at the horizon. The lowest value of this tension is equivalent to τ_{QCD} .

3.2 Uniform rotation

Another interesting motion of the quark is the uniform rotation. According to [3, 4], here we use the cylindrical coordinates, $\tilde{X}^M = (t, \phi, \rho, x^3, z)$, namely, we transform

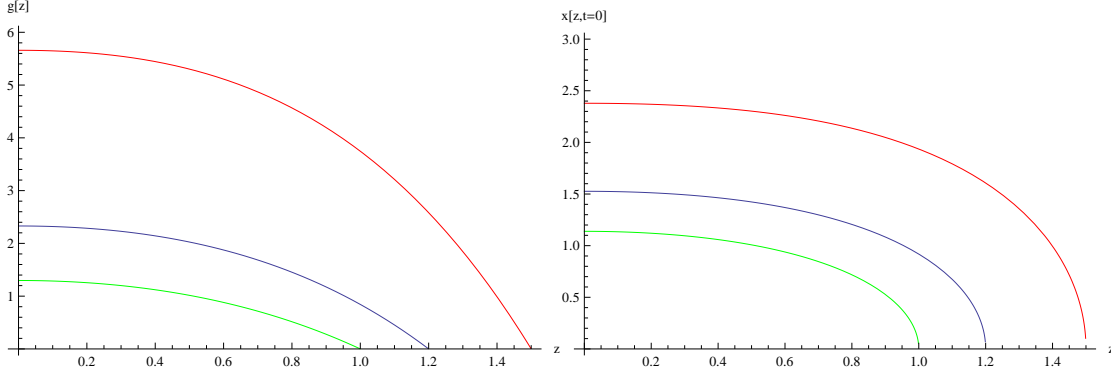


Fig. 2: Numerical plot of $g(z)$ and $x(z, t=0)$. ($\bar{z} = 1, 1.2, 1.5$) It is clear that $g(0) = 1/a^2$ increases with \bar{z} .

3-space coordinates (x^1, x^2, x^3) to the cylindrical coordinates (ϕ, ρ, x^3) as

$$x^1 = \rho \cos \phi, \quad x^2 = \rho \sin \phi, \quad x^3 = x^3.$$

In the cylindrical coordinates system, the metric (4) is rewritten as

$$ds^2 = e^{\Phi/2} \frac{L^2}{z^2} \left(-dt^2 + d\rho^2 + \rho^2 d\phi^2 + (dx^3)^2 + dz^2 \right), \quad e^{\Phi} = 1 + qz^4.$$

The ansatz for a rotating string is given by

$$\begin{aligned} \tilde{X}^M &= (t, \phi, \rho, x^3, z) \\ &= (t, \omega t + \theta(z), \rho(z), 0, z). \end{aligned}$$

Then the induced metric g_{ab} and the action S_{rot} become

$$g_{ab} = \begin{pmatrix} g_{tt} & g_{tz} \\ g_{zt} & g_{zz} \end{pmatrix} = e^{\Phi/2} \frac{L^2}{z^2} \begin{pmatrix} -1 + \omega^2 \rho^2 & \omega \theta' \rho^2 \\ \omega \theta' \rho^2 & 1 + \theta'^2 \rho^2 + \rho'^2 \end{pmatrix}, \quad (10)$$

$$S_{rot} = -\frac{1}{2\pi\alpha'} \int dt dz e^{\Phi/2} \frac{L^2}{z^2} \sqrt{(1 - \omega^2 \rho^2)(1 + \rho'^2) + \theta'^2 \rho^2}, \quad (11)$$

where the prime (\prime) denotes $\frac{\partial}{\partial z}$.

The equations of motion become

$$\partial_z \left\{ e^{\frac{\Phi}{2}} \frac{L^2}{z^2} \frac{(1 - \omega^2 \rho^2) \partial_z \rho}{\sqrt{(1 - \omega^2 \rho^2)(1 + (\partial_z \rho)^2) + \rho^2 (\partial_z \theta)^2}} \right\} = e^{\frac{\Phi}{2}} \frac{L^2}{z^2} \frac{-\omega^2 \rho (1 + (\partial_z \rho)^2) + \rho (\partial_z \theta)^2}{\sqrt{(1 - \omega^2 \rho^2)(1 + (\partial_z \rho)^2) + \rho^2 (\partial_z \theta)^2}}, \quad (12)$$

for ρ ,

$$e^{\frac{\Phi}{2}} \frac{L^2}{z^2} \frac{\rho^2 \partial_z \theta}{\sqrt{1 - \omega^2 \rho^2 + (1 - \omega^2 \rho^2) (\partial_z \rho)^2 + \rho^2 (\partial_z \theta)^2}} = \Pi, \quad (13)$$

for ϕ , where Π is the integration constant.

Solution We can easily solve Eq. (13) respect to $\partial_z \theta$. The solution is

$$\partial_z \theta = \frac{1}{\rho} \sqrt{\frac{\Pi^2 (1 - \omega^2 \rho^2) (1 + (\partial_z \rho)^2)}{\left(e^{\frac{\Phi}{2}} \frac{1}{z^2}\right)^2 \rho^2 - \Pi^2}}, \quad (14)$$

The fact that Eq. (14) must be real requires the following conditions,

$$\rho(\bar{z}) = \frac{1}{\omega}, \quad (15)$$

$$\frac{1}{\Pi \omega} = \frac{\bar{z}^2}{e^{\frac{\Phi}{2}}}. \quad (16)$$

Since (tt) -component of the induced metric (10) is

$$g_{tt} = e^{\frac{\Phi}{2}} \frac{L^2}{z^2} (-1 + \omega^2 \rho^2),$$

$z = \bar{z}$ agree with the position of the induced metric horizon. Solving the condition (16), we have

$$\bar{z}^4 = \frac{1}{\Pi^2 \omega^2 - q} > 0. \quad (17)$$

This value must be positive because \bar{z} is real number. The condition gives a constraint among the parameters, Π, ω and q ,

$$\Pi^2 \omega^2 - q > 0. \quad (18)$$

Substituting the equation (14) into the equation of motion (12), we have the differential equation,

$$\partial_z^2 \rho + \left(\frac{z(1 + qz^4)\rho + 2\rho^2 \partial_z \rho}{z(z^4 \Pi^2 - (1 + qz^4)\rho^2)} + \frac{1}{\rho} \frac{1}{1 - \omega^2 \rho^2} \right) (1 + (\partial_z \rho)^2) = 0. \quad (19)$$

We can solve this equation numerically if we fix the boundary condition. However, we cannot choose boundary conditions freely because of the conditions (16). To understand how the boundary condition is determined, we expand the function $\rho(z)$ at $z = \bar{z}$,

$$\rho = \rho(\bar{z}) + \rho'(\bar{z})(z - \bar{z}) + \dots \quad (20)$$

Substitute (20) into (19) and solving the differential equation perturbatively, we have

$$\rho'(\bar{z}) = \frac{\sqrt{\bar{z}^2 \omega^2 (1 + q\bar{z}^4)^2 + 4 - \bar{z} \omega (1 + q\bar{z}^4)}}{2}. \quad (21)$$

So we must choose the boundary conditions as (15) and (21).

A typical example of the solution of the string configuration is shown in Fig. 3. The shape of the string in large z region with finite q is very different from that with $q = 0$ [3]. In AdS case ($q = 0$), $\rho(z)$ is proportional to z in large z limit, $\rho(z) \sim z$, and it is divergent linearly. In dAdS case ($q \neq 0$), $\rho(z)$ seems to be oscillating around a certain positive value with a finite amplitude keeping $\rho > 1/\omega$. (See Fig. 4.) We can find following feature in Fig. 4.

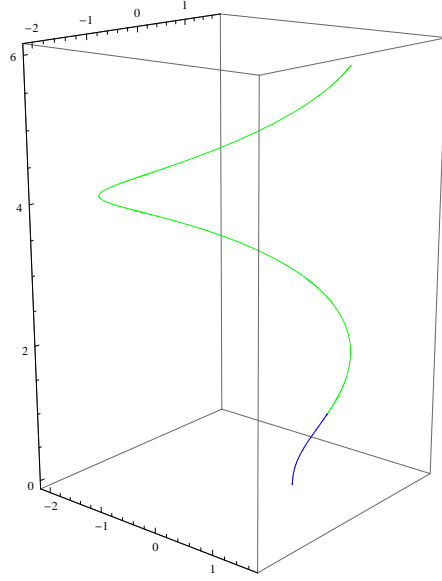


Fig. 3: A numerical plot of typical string configuration with $\Pi = 1$, $\omega = 1$ and $q = 0.1$. The lower [Blue] (upper [green]) line represents the outside (inside) of the induced metric horizon. The boundary is at $z = 0$ (the bottom of this graph.) The out side (the lower [blue]) part of the string is connected to the boundary causally but inside (the upper [green]) part is not. Therefore the energy flow to the inside of the induced metric horizon (the upper [green] part of the string) is identified with the ELR due to the radiation.

- For finite value of q , $\rho(z)$ oscillates with z . The period and the amplitude of the oscillation of $\rho(z)$ decrease with increasing q .
- The oscillation is not a simple trigonometric function.
- We cannot compute the solution in large value of q because of our computer performance. However the solutions cross the induced metric horizon ($\rho = 1/\omega$) only once in our calculations.
- In $q \rightarrow 0$ limit, we can confirm that the solution with finite q agree with that with $q = 0$.

The ELR for the uniform rotating quark is given by

$$\frac{dE}{dt} = \frac{1}{2\pi\alpha'} \frac{L^2}{\bar{z}^2} \sqrt{1 + q\bar{z}^4} = \frac{\omega\Pi L^2}{2\pi\alpha'}. \quad (22)$$

The lower bound arises from the constraint (18),

$$\frac{dE}{dt} > \frac{\sqrt{q}L^2}{2\pi\alpha'}. \quad (23)$$

It is very interesting that the lower bound (23) agree with the value of the lower bound of the ELR for uniform accelerating quark (8) in spite of the difference of the motions.

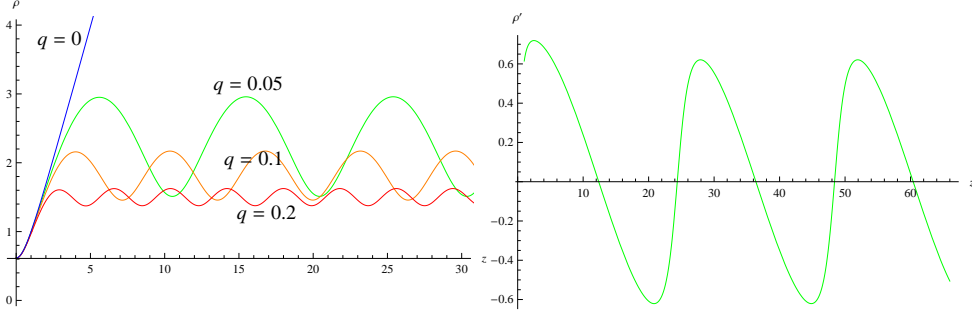


Fig. 4: Numerical plots of typical string configurations with $\Pi = 1$, $\omega = 1$. In the left, numerical plots of $\rho(z)$ with $q = 0$ (blue), $q = 0.05$ (green), $q = 0.1$ (orange) and $q = 0.2$ (red) are displayed. The period and amplitude decrease with increasing q . In the limit of $q \rightarrow 0$, the period and amplitude diverge and the solution approaches to that of the AdS. In the right, a numerical plot of $\rho'(z)$ with $q = 0.01$ is shown. We recognize that this oscillation cannot be described by simple trigonometric functions.

In this case also, we can understand this lower bound as τ_{QCD} from the minimum of the necessary force to pull the string for a unit time at the world sheet horizon. In the present case, the string can not be stretched to the centripetal force to keep the rotation of the quark as mentioned in the introduction. The string at the horizon should be pulled along the circle. As in the previous case, by using Eq. (11), we find

$$\frac{dE}{dt} = \frac{1}{2\pi\alpha'} \int_0^\omega d\theta \rho e^{\Phi/2} \frac{L^2}{z^2} = \frac{1}{2\pi\alpha'} e^{\Phi/2} \frac{L^2}{z^2} > \frac{\sqrt{q}L^2}{2\pi\alpha'}, \quad (24)$$

where $\rho\omega = 1$ and $z = \bar{z}$. Then we find again the same result with (22)

4 D4/ S^1 model

4.1 Linear acceleration(uniform acceleration)

We also investigate D4/ S^1 model. The string embedding function is parameterized as,

$$X^M = (t, x_1, x_2, x_3, x_4, u, \Omega_4) = (t, x(t, u), 0, 0, 0, u, \vec{0}).$$

Then induced metric g_{ab} , string action S and Π_u^t become

$$g_{ab} = \begin{pmatrix} -\left(\frac{u}{l}\right)^{3/2} (1 - \dot{x}^2) & \left(\frac{u}{l}\right)^{3/2} \dot{x}x' \\ \left(\frac{u}{l}\right)^{3/2} \dot{x}x' & \left(\frac{u}{l}\right)^{3/2} x'^2 + \left(\frac{l}{u}\right)^{3/2} f^{-1} \end{pmatrix}, \quad (25)$$

$$S = -\frac{1}{2\pi\alpha'} \sqrt{\left(\frac{u}{l}\right)^3 x'^2 + f^{-1} - \dot{x}^2 f^{-1}},$$

and

$$\Pi_t^u = \frac{1}{2\pi\alpha'} \frac{\left(\frac{u}{l}\right)^3 \dot{x}x'}{\sqrt{\left(\frac{u}{l}\right)^3 x'^2 + f^{-1} - \dot{x}^2 f^{-1}}}.$$

From Eq. (25), we realize that the induced metric horizon is found at the points (\bar{t}, \bar{u}) such that $\dot{x}(\bar{t}, \bar{u})^2 = 1$. Thus ELR becomes

$$\frac{dE}{dt} \equiv \Pi_t^u(\bar{t}, \bar{u}) = \frac{1}{2\pi\alpha'} \left(\frac{\bar{u}}{l}\right)^{3/2} > \frac{1}{2\pi\alpha'} \left(\frac{u_k}{l}\right)^{3/2}. \quad (26)$$

Because $u > u_k, \frac{dE}{dt}$ has the lower bound. It is considerable that the value of the lower bound (26) is coincide with the lower bound of the ELR of the rotating string (27). The situation is the same with the case of dAdS. We can understand this lower bound as the minimum of the necessary force to overcome the string tension, which is just $\frac{1}{2\pi\alpha'} \left(\frac{u_k}{l}\right)^{3/2} = \tau_{\text{QCD}}$ in this model. This is easily obtained through the calculation of the Wilson loop.

4.2 Uniform rotation

Because the detailed investigation of the rotating string in $D4/S^1$ model is already given by Ref. [4]. We just borrow the result from the work,

$$\frac{dE}{dt} = \frac{1}{2\pi\alpha'} \left(\frac{\bar{u}}{l}\right)^{3/2} > \frac{\Lambda_{QCD}^3}{2\pi\alpha'} = \frac{1}{2\pi\alpha'} \left(\frac{u_k}{l}\right)^{3/2}. \quad (27)$$

As expected, the ELR of the rotating quark has also the lower bound which agrees with (26).

5 Summary and discussion

We find the common lower bound of the ELR for two different accelerated motions of the quark, (8) and (23) or (26) and (27). The value is determined by the parameter q or u_k/l . It is given precisely by τ_{QCD} for each confinement model as expected

In dAdS model, the scale parameter q controls the dynamics. This parameter appears in various physical quantity. If we take the limit $q \rightarrow 0$, the metric of this model (4) reduces to the one of the AdS space and the lower bound of the energy loss late vanishes since $\tau_{\text{QCD}} \rightarrow 0$.

In $D4/S^1$ model, we can find the same reason why the energy loss has the lower bound. The lower bound of ELR is given by the tension τ_{QCD} of this model. Then, we may consider the present radiation as the jet of glueballs as pointed out in Ref. [4]. This assertion can be supported also by our result.

In both models, we found the common lower bounds of the ELR for two different accelerated motions of the quark. This point could be understood easily. In the models used here, the bulk metric G_{MN} is diagonal. Then, when a world sheet horizon exists in the embedded string configuration, the Eq. (3) can be derived without any other assumption for $X^M(\tau, \sigma)$. This implies that the Eq. (3) is available to various kinds of motion. [‡] Due to this fact, the results given by Eqs. (8), (26), (23), and (27) are all understood from Eq. (3).

[‡] We notice that the similar equation with (3) is also found in Ref. [9]

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